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REGION OF COMPARISON FOR THE SECOND ORDER MOVING AVERAGE AND PURE DIAGONAL BILINEAR PROCESSES

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ABSTRACT

The covariance structure among other properties of the pure diagonal bilinear time series process of order two is derived and compared with that of the linear moving average process of order two. Also obtained are the extrema of the autocorrelation functions of the two processes for the purpose of distinguishing between the competing models. The well known similarities in the covariance structures of pure diagonal bilinear model of order two and moving average process of order two is found to exist for certain intervals of autocorrelation coefficients of the models. Comparison of the models must be done at this common intervals.

KEYWORDS: Covariance Structures, Pure Diagonal Bilinear Time Series Model, Linear Moving Average Process, Extrema of Autocorrelation Coeffiecints

2010hematics Subject Classification: 62M10

1. INTRODUCTION

Many non stationary time series have features which can not be appropriately explained by a linear model (Bibi and Oyet, 2002). This limitation of linear time series models led to the development of non linear time series models. A special class of non linear models that has gained acceptability of analysts is the bilinear model (Li (1992) and Subba Rao and Da Silva (1992)). Bilinear models are generally classified into subdiagonal, pure diagonal and superdiagonal bilinear models. Authors' preference for the pure diagonal bilinear among the three types of bilinear models, has been attributed to its simple mathematical structure, existence and invertibility conditions. A stochastic process $X_t, t \in T$ is called a pure diagonal bilinear time series process of order p if

$$X_t = \sum_{i=1}^{p} \beta_i X_{t-i} e_{t-i} + e_t \tag{1.1}$$

where $\beta_1, \beta_2, ..., \beta_p$ are real constants and $e_t, e_{t-1}, ..., e_{t-p}$ is a sequence of independent identically distributed random variables with zero mean and constant variance $\sigma_1^{\ 2}$. Following the results in Khadija et al (2007) and Liu (1990), the model (1.1) is invertible if

$$\sum_{i=1}^{p} \beta_i^{\ 2} \sigma_1^{\ 2} < p^{-2} \tag{1.2}$$

Again, it can be deduced from Oyet (2001) that the model (1.1) is stationary if

$$\sum_{i=1}^{p} \beta_i^{\,2} \sigma_1^{\,2} < 1 \tag{1.3}$$

Granger and Andersen, 1978, Subba Rao, 1981 and Akamanam, et al.(1986) established that model (1.1) competes with the non zero mean moving average process of order p due to their similar covariance structures.

Let $Y_t, t \in Z$ be a a stochastic process. If $\mu_t, t \in Z$ is a white noise process with zero mean and constant variance $(\sigma_2^{\,2})$, then $Y_t, t \in T$ is called a non zero mean moving average process of order 2 (see Hamilton, 1994) if the following equation is satisfied:

$$Y_t = \theta_0 + \theta_1 \mu_{t-1} + \theta_2 \mu_{t-2} + \mu_t \tag{1.4}$$

Suppose ρ_1 and ρ_2 represent the first and second order autocorrelation coefficients of model (1.4). Then ρ_1 attains a minimum and a maximum values of -0.71 and 0.71 respectively. On the other hand, ρ_2 has a minimum and a maximum value of -0.5 and 0.5 respectively (Okereke et al, 2012).

Authors have proposed several methods of differentiating between the pure diagonal bilinear process and moving average process. One of these techniques has to do with the computation of the third order moment and cumulant (Martins, 1999, Iwueze and Chikezie, 2006). A major pitfall of this approach is that it is usually difficult to compute and interprete third moments and cumulants. In view of this fact, Palma and zevallos(2001), Omekara(2010) and Iwueze and Ohakwe (2011) used the covariance structure of the squares of the time series data to distinguish between the competing models. Suffice it to say that the aforementioned methods do not spell out when it is necessary to consider a test for linearity of a given time series. Moreover, they are all time consuming in the sense that they require more computations are required than covariance analysis. To overcome these problems, Ohakwe and Iwueze(2009) used a new method that depends on the extrema of the autocorrelation coefficients to distinguish between PDB(1) and MA(1). In this study, the autocorrelation coefficients of PDB(2) and MA(2) are examined with a view to differentiating between the competing models under covariance analysis.

2. PURE DIAGONAL BILINEAR PROCESS OF ORDER TWO

When $p = 2$ is substituted in model(1.1), we obtain a pure diagonal bilinear time series model of order two given as

$$X_t = \beta_1 X_{t-1} e_{t-1} + \beta_2 X_{t-2} e_{t-2} + e_t \tag{2.1}$$

where $e_t, t \in T$ is a sequence of independent identically distributed random variables with mean 0 and constant variance $\sigma_1^{\,2} < \infty$.

It can be shown that the first order moment of (2.1) is :

$$E(X_t) = (\lambda_1 + \lambda_2)\sigma_1 \tag{2.2}$$

The second order moments of (2.1) are easily obtained to be

$$E(X_t X_{t-k}) = \begin{cases} \dfrac{\sigma_1^{\,2}(1+2\lambda_1\lambda_2+2\lambda_1^{\,2}+2\lambda_2^{\,2})}{1-\lambda_1^{\,2}-\lambda_2^{\,2}}, k=0 \\ \dfrac{\sigma_1^{\,2}\begin{bmatrix} 2\lambda_1^{\,2}-2\lambda_1^{\,4}-2\lambda_1^{\,2}\lambda_2^{\,2}+5\lambda_1\lambda_2 \\ -2\lambda_1^{\,3}\lambda_2-2\lambda_1\lambda_2^{\,3}+\lambda_2^{\,2}-\lambda_2^{\,4}\end{bmatrix}}{1-\theta_1^{\,2}\sigma_2^{\,2}-\theta_2^{\,2}\sigma_2^{\,2}}, k=\pm 1 \\ \sigma_2^{\,2}(\lambda_1^{\,2}+3\lambda_1\lambda_2+2\lambda_2^{\,2}), k=\pm 2 \\ \sigma_2^{\,2}(\lambda_1^{\,2}+2\lambda_1\lambda_2+\lambda_2^{\,2}), k\geq 3 \end{cases} \tag{2.3}$$

Using the fact that

$$R(k) = Cov(X_t X_{t-k}) = \mathrm{E}(\mathrm{X_t} X_{t-k}) - \mu_x^{\,2} \tag{2.4}$$

and

$$\rho_k = \frac{R(k)}{R(0)} \tag{2.5}$$

we obtain

$$R(k) = \begin{cases} \dfrac{\sigma_2^{\,2}(1+\lambda_1^{\,2}+\lambda_2^{\,2}+\lambda_1^{\,4}+2\lambda_1^{\,2}\lambda_2^{\,2}+2\lambda_1^{\,3}\lambda_2+2\lambda_1\lambda_2^{\,3}+\lambda_2^{\,4})}{1-\lambda_1^{\,2}-\lambda_2^{\,2}}, k=0 \\ \dfrac{\sigma_2^{\,2}(\lambda_1^{\,2}-\lambda_1^{\,4}+\lambda_1^{\,2}\lambda_2^{\,2}+3\lambda_1\lambda_2)}{1-\lambda_1^{\,2}-\lambda_2^{\,2}}, k=\pm 1 \\ \sigma_2^{\,2}(\lambda_1\lambda_2+\lambda_2^{\,2}), k=\pm 2 \\ 0, k\geq 3 \end{cases} \tag{2.6}$$

And

$$\rho_k = \begin{cases} 1, k=0 \\ \dfrac{\lambda_1^{\,2}-\lambda_1^{\,4}+\lambda_1^{\,2}\lambda_2^{\,2}+3\lambda_1\lambda_2}{1+\lambda_1^{\,2}+\lambda_2^{\,2}+\lambda_1^{\,4}+2\lambda_1^{\,2}\lambda_2^{\,2}+2\lambda_1^{\,3}\lambda_2+2\lambda_1\lambda_2^{\,3}+\lambda_2^{\,4}}, k=\pm 1 \\ \dfrac{(\lambda_1\lambda_2+\lambda_2^{\,2})(1-\lambda_1^{\,2}-\lambda_2^{\,2})}{1+\lambda_1^{\,2}+\lambda_2^{\,2}+\lambda_1^{\,4}+2\lambda_1^{\,2}\lambda_2^{\,2}+2\lambda_1^{\,3}\lambda_2+2\lambda_1\lambda_2^{\,3}+\lambda_2^{\,4}}, k=\pm 2 \\ 0, k\geq 3 \end{cases} \tag{2.7}$$

where $\lambda_1 = \beta_1\sigma_1$ and $\lambda_2 = \beta_2\sigma_1$.

It then follows from (2.6) that the second order moment of X_t exists if $\lambda_1^{\,2}+\lambda_2^{\,2} < 1$. To recover the errors from the observed values of X_t and also to make proper forecast using (2.1), the model has to be invertible. Using (1.2), the model in (2.1) is invertible if

$$\lambda_1^{\,2}+\lambda_2^{\,2} < 0.25 \tag{2.8}$$

Figure 1 shows the stationarity and invertibility region for the PDB (2) process

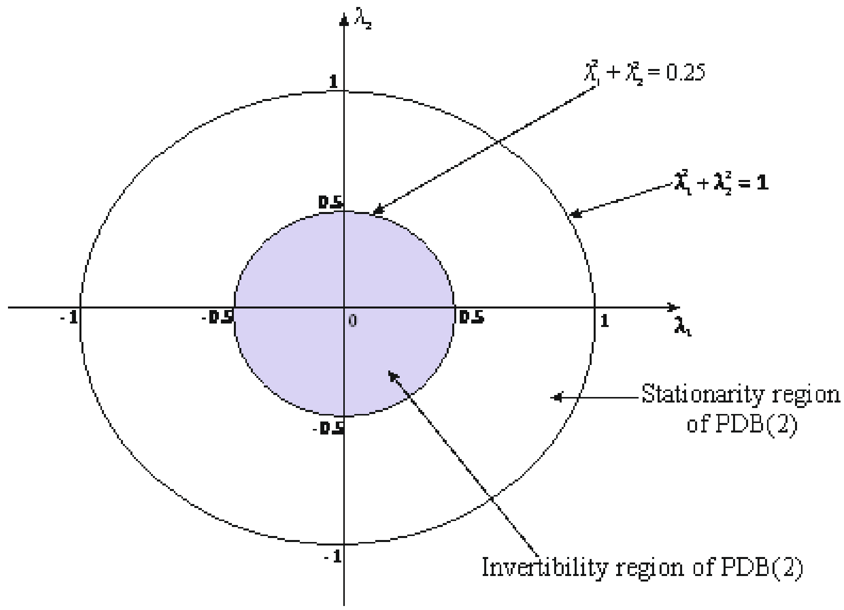

Figure 1: The Stationarity and Invertibility Regions of the PDB (2) Process

3. MINIMUM AND MAXIMUM VALUES OF ρ_1 AND ρ_2 FOR PURE DIAGONAL BILINEAR ROCESS OF ORDER TWO

We can observe from Figure 1 that the set of values satisfying the invertibility condition (2.1) also satisfy its stationarity condition. Consequently, the first order and second order autocorrelation coefficients of PDB(2) are computed based on 7822 values of λ_1 and λ_2 for which the invertibility condition, $\lambda_1^2 + \lambda_2^2 < 0.25$ is satisfied. For want of space, 580 of this set of the computations are shown in Table 1.

Table 1: ρ_1 and ρ_2 for Selected λ_1 and λ_2 in the Stationarity and Invertibilty Regions of PDB (2)

S/No	λ_1	λ_2	ρ_1	ρ_2	S/ No	λ_1	λ_2	ρ_1	ρ_2	S/No	λ_1	λ_2	ρ_1	ρ_2
1	-0.49	0.09	0.04	-0.02	35	-0.48	0.02	0.12	-0.01	69	0.07	-0.02	0.07	-0.02
2	-0.49	0.08	0.05	-0.02	36	-0.48	0.03	0.11	-0.01	70	0.06	-0.02	0.06	-0.02
3	-0.49	0.07	0.06	-0.02	37	-0.48	0.04	0.09	-0.01	71	0.05	-0.02	0.05	-0.02
4	-0.49	0.06	0.07	-0.02	38	-0.48	0.05	0.08	-0.01	72	0.04	-0.02	0.04	-0.02
5	-0.49	0.05	0.08	-0.01	39	-0.48	0.06	0.07	-0.02	73	0.03	-0.02	0.03	-0.02
6	-0.49	0.04	0.1	-0.01	40	-0.48	0.07	0.06	-0.02	74	0.02	-0.02	0.02	-0.02
7	-0.49	0.03	0.11	-0.01	41	-0.48	0.08	0.05	-0.02	75	0.00	-0.03	0.00	-0.03
8	-0.49	0.02	0.12	-0.01	42	-0.48	0.09	0.04	-0.02	76	-0.01	-0.03	-0.01	-0.03
9	-0.49	0	0.14	0	43	-0.48	0.1	0.03	-0.02	77	-0.02	-0.03	-0.02	-0.03
10	-0.49	0	0.14	0	44	-0.48	0.11	0.02	-0.02	78	-0.03	-0.03	-0.03	-0.03
11	-0.49	-0.01	0.15	0	45	-0.48	0.12	0.01	-0.03	79	-0.04	-0.03	-0.04	-0.03
12	-0.49	-0.02	0.16	0.01	46	-0.48	0.13	0	-0.03	80	0.32	0.07	0.32	0.07
13	-0.49	-0.03	0.17	0.01	47	-0.47	-0.16	0.30	0.06	81	0.31	0.06	0.31	0.06
14	-0.49	-0.04	0.18	0.01	48	-0.47	-0.15	0.29	0.05	82	0.31	0.06	0.31	0.06
15	-0.49	-0.05	0.2	0.02	49	-0.47	-0.14	0.28	0.05	83	0.30	0.06	0.30	0.06
16	-0.49	-0.06	0.21	0.02	50	-0.47	-0.13	0.27	0.04	84	0.29	0.05	0.29	0.05
17	-0.49	-0.07	0.22	0.02	51	0.26	0.04	0.26	0.04	85	0.28	0.05	0.28	0.05
18	-0.49	-0.08	0.23	0.03	52	0.25	0.04	0.25	0.04	86	0.27	0.05	0.27	0.05
19	-0.49	-0.09	0.24	0.03	53	0.24	0.03	0.24	0.03	87	0.26	0.04	0.26	0.04

Table 1 – Contd.,

S/No	λ_1	λ_2	ρ_1	ρ_2	S/ No	λ_1	λ_2	ρ_1	ρ_2	S/No	λ_1	λ_2	ρ_1	ρ_2
20	-0.48	-0.13	0.28	0.04	54	0.23	0.03	0.23	0.03	88	0.25	0.04	0.25	0.04
21	-0.48	-0.12	0.27	0.04	55	0.22	0.03	0.22	0.03	89	0.24	0.03	0.24	0.03
22	-0.48	-0.11	0.26	0.04	56	0.21	0.02	0.21	0.02	90	0.23	0.03	0.23	0.03
23	-0.48	-0.1	0.24	0.03	57	0.20	0.02	0.20	0.02	91	0.22	0.03	0.22	0.03
24	-0.48	-0.09	0.23	0.03	58	0.19	0.02	0.19	0.02	92	0.21	0.02	0.21	0.02
25	-0.48	-0.08	0.22	0.03	59	0.18	0.01	0.18	0.01	93	0.20	0.02	0.20	0.02
26	-0.48	-0.07	0.21	0.02	60	0.17	0.01	0.17	0.01	94	-0.46	-0.05	0.19	0.02
27	-0.48	-0.06	0.2	0.02	61	0.16	0.01	0.16	0.01	95	-0.46	-0.04	0.18	0.01
28	-0.48	-0.05	0.19	0.02	62	0.15	0.00	0.15	0.00	96	-0.46	-0.03	0.16	0.01
29	-0.48	-0.04	0.18	0.01	63	0.14	0.00	0.14	0.00	97	-0.46	-0.02	0.15	0.01
30	-0.48	-0.03	0.17	0.01	64	0.12	-0.00	0.12	-0.00	98	-0.46	-0.01	0.14	0.00
31	-0.48	-0.02	0.16	0.01	65	0.11	-0.01	0.11	-0.01	99	-0.46	0.00	0.13	0.00
32	-0.48	-0.01	0.15	0	66	0.10	-0.01	0.10	-0.01	100	-0.46	0.01	0.12	-0.00
33	-0.48	0	0.14	0	67	0.09	-0.01	0.09	-0.01	101	-0.46	0.02	0.11	-0.01
34	-0.48	0.01	0.13	0	68	0.08	-0.01	0.08	-0.01	102	-0.46	0.03	0.10	-0.01
103	-0.46	0.04	0.09	-0.01	151	-0.45	0.11	0.01	-0.02	198	-0.44	0.13	-0.01	-0.03
104	-0.46	0.05	0.08	-0.01	152	-0.45	0.12	0.00	-0.02	199	-0.44	0.14	-0.02	-0.03
105	-0.46	0.06	0.07	-0.02	153	-0.45	0.13	-0.01	-0.03	200	-0.44	0.15	-0.03	-0.03
106	-0.46	0.07	0.06	-0.02	154	-0.45	0.14	-0.02	-0.03	201	-0.44	0.16	-0.04	-0.03
107	-0.46	0.08	0.05	-0.02	155	-0.45	0.15	-0.03	-0.03	202	-0.44	0.17	-0.05	-0.03
108	-0.46	0.09	0.04	-0.02	156	-0.45	0.16	-0.04	-0.03	203	-0.44	0.18	-0.06	-0.03
109	-0.46	0.10	0.02	-0.02	157	-0.45	0.17	-0.05	-0.03	204	-0.44	0.19	-0.07	-0.03
110	-0.46	0.11	0.01	-0.02	158	-0.45	0.18	-0.06	-0.03	205	-0.44	0.20	-0.08	-0.03
111	-0.46	0.12	0.00	-0.03	159	-0.45	0.19	-0.07	-0.03	206	-0.44	0.21	-0.09	-0.03
112	-0.46	0.13	-0.01	-0.03	160	-0.45	0.20	-0.08	-0.03	207	-0.44	0.22	-0.10	-0.03
113	-0.46	0.14	-0.02	-0.03	161	-0.45	0.21	-0.09	-0.03	208	-0.44	0.23	-0.11	-0.03
114	-0.46	0.15	-0.03	-0.03	162	-0.44	-0.23	0.35	0.09	209	-0.43	-0.25	0.36	0.09
115	-0.46	0.16	-0.04	-0.03	163	-0.44	-0.22	0.34	0.08	210	-0.43	-0.24	0.35	0.09
116	-0.46	0.17	-0.05	-0.03	164	-0.44	-0.21	0.33	0.08	211	-0.43	-0.23	0.34	0.09
117	-0.46	0.18	-0.06	-0.03	165	-0.44	-0.20	0.32	0.07	212	-0.43	-0.22	0.33	0.08
119	-0.46	0.19	-0.07	-0.03	166	-0.44	-0.19	0.31	0.07	213	-0.43	-0.21	0.32	0.08
120	-0.45	-0.21	0.34	0.08	167	-0.44	-0.18	0.30	0.07	214	-0.43	-0.20	0.32	0.07
121	-0.45	-0.20	0.33	0.07	168	-0.44	-0.17	0.30	0.06	215	-0.43	-0.19	0.31	0.07
122	-0.45	-0.19	0.32	0.07	169	-0.44	-0.16	0.29	0.06	216	-0.43	-0.18	0.30	0.07
123	-0.45	-0.18	0.31	0.07	170	-0.44	-0.15	0.28	0.05	217	-0.43	-0.17	0.29	0.06
124	-0.45	-0.17	0.30	0.06	171	-0.44	-0.14	0.27	0.05	218	-0.43	-0.16	0.28	0.06
125	-0.45	-0.16	0.29	0.06	172	-0.44	-0.13	0.26	0.05	219	-0.43	-0.15	0.27	0.05
126	-0.45	-0.15	0.28	0.05	173	-0.44	-0.12	0.25	0.04	220	-0.43	-0.14	0.26	0.05
127	-0.45	-0.14	0.27	0.05	174	-0.44	-0.11	0.24	0.04	221	-0.43	-0.13	0.25	0.05
128	-0.45	-0.13	0.26	0.05	175	-0.44	-0.10	0.23	0.03	222	-0.43	-0.12	0.24	0.04
129	-0.45	-0.12	0.25	0.04	176	-0.44	-0.09	0.22	0.03	223	-0.43	-0.11	0.24	0.04
130	-0.45	-0.11	0.24	0.04	177	-0.44	-0.08	0.21	0.03	224	-0.43	-0.10	0.23	0.03
131	-0.45	-0.09	0.22	0.03	178	-0.44	-0.07	0.20	0.02	225	-0.43	-0.09	0.22	0.03
132	-0.45	-0.08	0.21	0.03	179	-0.44	-0.06	0.19	0.02	226	-0.43	-0.08	0.21	0.03
133	-0.45	-0.07	0.20	0.02	180	-0.44	-0.05	0.18	0.02	227	-0.43	-0.07	0.20	0.02
134	-0.45	-0.06	0.19	0.02	181	-0.44	-0.04	0.17	0.01	228	-0.43	-0.06	0.19	0.02
135	-0.45	-0.05	0.18	0.02	182	-0.44	-0.03	0.16	0.01	229	-0.43	-0.05	0.18	0.02
136	-0.45	-0.04	0.17	0.01	183	-0.44	-0.02	0.15	0.01	230	-0.43	-0.04	0.17	0.01
137	-0.45	-0.03	0.16	0.01	184	-0.44	-0.01	0.14	0.00	231	-0.43	-0.03	0.15	0.01
138	-0.45	-0.02	0.15	0.01	185	-0.44	0.00	0.13	0.00	232	-0.43	-0.02	0.14	0.01
139	-0.45	-0.01	0.14	0.00	186	-0.44	0.01	0.12	-0.00	233	-0.43	-0.01	0.13	0.00
140	-0.45	0.00	0.13	0.00	187	-0.44	0.02	0.11	-0.01	234	-0.43	0.00	0.12	0.00
141	-0.45	0.01	0.12	-0.00	188	-0.44	0.03	0.10	-0.01	235	-0.43	0.01	0.11	-0.00
142	-0.45	0.02	0.11	-0.01	189	-0.44	0.04	0.08	-0.01	236	-0.43	0.02	0.10	-0.01
143	-0.45	0.03	0.10	-0.01	190	-0.44	0.05	0.07	-0.01	237	-0.43	0.03	0.09	-0.01
144	-0.45	0.04	0.09	-0.01	191	-0.44	0.06	0.06	-0.01	238	-0.43	0.04	0.08	-0.01
145	-0.45	0.05	0.08	-0.01	192	-0.44	0.07	0.05	-0.02	239	-0.43	0.05	0.07	-0.01
146	-0.45	0.06	0.07	-0.02	193	-0.44	0.08	0.04	-0.02	240	-0.43	0.06	0.06	-0.01
147	-0.45	0.07	0.05	-0.02	194	-0.44	0.09	0.03	-0.02	241	-0.43	0.07	0.05	-0.02
148	-0.45	0.08	0.04	-0.02	195	-0.44	0.10	0.02	-0.02	242	-0.43	0.08	0.04	-0.02
149	-0.45	0.09	0.03	-0.02	196	-0.44	0.11	0.01	-0.02	243	-0.43	0.09	0.03	-0.02
150	-0.45	0.10	0.02	-0.02	197	-0.44	0.12	0.00	-0.02	244	-0.43	0.10	0.02	-0.02

Table 1 – Contd.,

S/No	λ_1	λ_2	ρ_1	ρ_2	S/ No	λ_1	λ_2	ρ_1	ρ_2	S/No	λ_1	λ_2	ρ_1	ρ_2
245	-0.43	0.11	0.01	-0.02	293	-0.42	0.06	0.06	-0.01	341	-0.41	-0.02	0.14	0.01
246	-0.43	0.12	-0.00	-0.02	294	-0.42	0.07	0.05	-0.02	342	-0.41	-0.01	0.13	0.00
247	-0.43	0.13	-0.01	-0.03	295	-0.42	0.08	0.04	-0.02	343	-0.41	0.00	0.12	0.00
248	-0.43	0.14	-0.02	-0.03	296	-0.42	0.09	0.03	-0.02	344	-0.41	0.01	0.11	-0.00
249	-0.43	0.15	-0.03	-0.03	297	-0.42	0.10	0.02	-0.02	345	-0.41	0.02	0.10	-0.01
250	-0.43	0.16	-0.04	-0.03	298	-0.42	0.11	0.01	-0.02	346	-0.41	0.03	0.09	-0.01
251	-0.43	0.17	-0.05	-0.03	299	-0.42	0.12	-0.00	-0.02	347	-0.41	0.04	0.08	-0.01
252	-0.43	0.18	-0.06	-0.03	300	-0.42	0.13	-0.01	-0.03	348	-0.41	0.05	0.07	-0.01
253	-0.43	0.19	-0.07	-0.03	301	-0.42	0.14	-0.02	-0.03	349	-0.41	0.06	0.06	-0.01
254	-0.43	0.20	-0.08	-0.03	302	-0.42	0.15	-0.03	-0.03	350	-0.41	0.07	0.05	-0.02
255	-0.43	0.21	-0.09	-0.03	303	-0.42	0.16	-0.04	-0.03	351	-0.41	0.08	0.04	-0.02
256	-0.43	0.22	-0.10	-0.03	304	-0.42	0.17	-0.05	-0.03	352	-0.41	0.09	0.03	-0.02
257	-0.43	0.23	-0.11	-0.03	305	-0.42	0.18	-0.06	-0.03	353	-0.41	0.10	0.02	-0.02
258	-0.43	0.24	-0.12	-0.03	306	-0.42	0.19	-0.07	-0.03	354	-0.41	0.11	0.01	-0.02
259	-0.43	0.25	-0.13	-0.03	307	-0.42	0.20	-0.08	-0.03	355	-0.41	0.12	-0.00	-0.02
260	-0.42	-0.27	0.36	0.10	308	-0.42	0.21	-0.09	-0.03	356	-0.41	0.13	-0.01	-0.02
261	-0.42	-0.26	0.36	0.10	309	-0.42	0.22	-0.10	-0.03	357	-0.41	0.14	-0.02	-0.03
262	-0.42	-0.25	0.35	0.09	310	-0.42	0.23	-0.11	-0.03	358	-0.41	0.15	-0.03	-0.03
263	-0.42	-0.24	0.34	0.09	311	-0.42	0.24	-0.12	-0.03	359	-0.41	0.16	-0.04	-0.03
264	-0.42	-0.23	0.34	0.09	312	-0.42	0.25	-0.13	-0.03	360	-0.41	0.17	-0.05	-0.03
265	-0.42	-0.22	0.33	0.08	313	-0.42	0.26	-0.14	-0.03	361	-0.41	0.18	-0.06	-0.03
266	-0.42	-0.21	0.32	0.08	314	-0.42	0.27	-0.15	-0.02	362	-0.41	0.19	-0.07	-0.03
267	-0.42	-0.20	0.31	0.07	315	-0.41	-0.28	0.36	0.11	363	-0.41	0.20	-0.08	-0.03
268	-0.42	-0.19	0.30	0.07	316	-0.41	-0.27	0.36	0.10	364	-0.41	0.21	-0.09	-0.03
269	-0.42	-0.18	0.29	0.07	317	-0.41	-0.26	0.35	0.10	365	-0.41	0.22	-0.10	-0.03
270	-0.42	-0.17	0.29	0.06	318	-0.41	-0.25	0.34	0.10	366	-0.41	0.23	-0.11	-0.03
271	-0.42	-0.16	0.28	0.06	319	-0.41	-0.24	0.34	0.09	367	-0.41	0.24	-0.12	-0.03
272	-0.42	-0.15	0.27	0.05	320	-0.41	-0.23	0.33	0.09	368	-0.41	0.25	-0.13	-0.02
273	-0.42	-0.14	0.26	0.05	321	-0.41	-0.22	0.32	0.08	369	-0.41	0.26	-0.14	-0.02
274	-0.42	-0.13	0.25	0.05	322	-0.41	-0.21	0.31	0.08	370	-0.41	0.27	-0.14	-0.02
275	-0.42	-0.12	0.24	0.04	323	-0.41	-0.20	0.31	0.08	371	-0.41	0.28	-0.15	-0.02
276	-0.42	-0.11	0.23	0.04	324	-0.41	-0.19	0.30	0.07	372	-0.40	-0.29	0.36	0.11
277	-0.42	-0.10	0.22	0.03	325	-0.41	-0.18	0.29	0.07	373	-0.40	-0.28	0.36	0.11
278	-0.42	-0.09	0.21	0.03	326	-0.41	-0.17	0.28	0.06	374	-0.40	-0.27	0.35	0.10
279	-0.42	-0.08	0.20	0.03	327	-0.41	-0.16	0.27	0.06	375	-0.40	-0.26	0.34	0.10
280	-0.42	-0.07	0.19	0.02	328	-0.41	-0.15	0.26	0.05	376	-0.40	-0.25	0.34	0.10
281	-0.42	-0.06	0.18	0.02	329	-0.41	-0.14	0.25	0.05	377	-0.40	-0.24	0.33	0.09
282	-0.42	-0.05	0.17	0.02	330	-0.41	-0.13	0.24	0.05	378	-0.40	-0.23	0.32	0.09
283	-0.42	-0.04	0.16	0.01	331	-0.41	-0.12	0.23	0.04	379	-0.40	-0.22	0.32	0.08
284	-0.42	-0.03	0.15	0.01	332	-0.41	-0.11	0.23	0.04	380	-0.40	-0.21	0.31	0.08
285	-0.42	-0.02	0.14	0.01	333	-0.41	-0.10	0.22	0.03	381	-0.40	-0.20	0.30	0.08
286	-0.42	-0.01	0.13	0.00	334	-0.41	-0.09	0.21	0.03	382	-0.40	-0.19	0.29	0.07
287	-0.42	0.00	0.12	0.00	335	-0.41	-0.08	0.20	0.03	383	-0.40	-0.18	0.28	0.07
288	-0.42	0.01	0.11	-0.00	336	-0.41	-0.07	0.19	0.02	384	-0.40	-0.17	0.27	0.06
289	-0.42	0.02	0.10	-0.01	337	-0.41	-0.06	0.18	0.02	385	-0.40	-0.16	0.27	0.06
290	-0.42	0.03	0.09	-0.01	338	-0.41	-0.05	0.17	0.02	386	-0.40	-0.15	0.26	0.05
291	-0.42	0.04	0.08	-0.01	339	-0.41	-0.04	0.16	0.01	387	-0.40	-0.14	0.25	0.05
292	-0.42	0.05	0.07	-0.01	340	-0.41	-0.03	0.15	0.01	388	-0.40	-0.13	0.24	0.05
389	-0.40	-0.12	0.23	0.04	439	-0.39	-0.23	0.32	0.09	489	-0.39	0.27	-0.14	-0.02
390	-0.40	-0.11	0.22	0.04	440	-0.39	-0.22	0.31	0.08	490	-0.39	0.28	-0.15	-0.02
391	-0.40	-0.10	0.21	0.03	441	-0.39	-0.21	0.30	0.08	491	-0.39	0.29	-0.16	-0.02
392	-0.40	-0.09	0.20	0.03	442	-0.39	-0.20	0.29	0.08	492	-0.39	0.30	-0.17	-0.02
393	-0.40	-0.08	0.19	0.03	443	-0.39	-0.19	0.29	0.07	493	-0.39	0.31	-0.18	-0.01
394	-0.40	-0.07	0.18	0.02	444	-0.39	-0.18	0.28	0.07	494	-0.38	-0.32	0.37	0.12
395	-0.40	-0.06	0.17	0.02	445	-0.39	-0.17	0.27	0.06	495	-0.38	-0.31	0.36	0.12
396	-0.40	-0.05	0.16	0.02	446	-0.39	-0.16	0.26	0.06	496	-0.38	-0.30	0.36	0.12
397	-0.40	-0.04	0.15	0.01	447	-0.39	-0.15	0.25	0.05	497	-0.38	-0.29	0.35	0.11
398	-0.40	-0.03	0.14	0.01	448	-0.39	-0.14	0.24	0.05	498	-0.38	-0.28	0.34	0.11
399	-0.40	-0.02	0.13	0.01	449	-0.39	-0.13	0.23	0.05	499	-0.38	-0.27	0.34	0.10
400	-0.40	-0.01	0.12	0.00	450	-0.39	-0.12	0.22	0.04	500	-0.38	-0.26	0.33	0.10
401	-0.40	0.00	0.11	0.00	451	-0.39	-0.11	0.22	0.04	501	-0.38	-0.25	0.32	0.10
402	-0.40	0.01	0.10	-0.00	452	-0.39	-0.10	0.21	0.03	502	-0.38	-0.24	0.32	0.09

Table 1 – Contd.,

S/No	λ_1	λ_2	ρ_1	ρ_2	S/ No	λ_1	λ_2	ρ_1	ρ_2	S/No	λ_1	λ_2	ρ_1	ρ_2
403	-0.40	0.02	0.09	-0.01	453	-0.39	-0.09	0.20	0.03	503	-0.38	-0.23	0.31	0.09
404	-0.40	0.03	0.08	-0.01	454	-0.39	-0.08	0.19	0.03	504	-0.38	-0.22	0.30	0.08
405	-0.40	0.04	0.07	-0.01	455	-0.39	-0.07	0.18	0.02	505	-0.38	-0.21	0.29	0.08
406	-0.40	0.05	0.06	-0.01	456	-0.39	-0.06	0.17	0.02	506	-0.38	-0.20	0.29	0.08
407	-0.40	0.06	0.05	-0.01	457	-0.39	-0.05	0.16	0.02	507	-0.38	-0.19	0.28	0.07
408	-0.40	0.07	0.04	-0.02	458	-0.39	-0.04	0.15	0.01	508	-0.38	-0.18	0.27	0.07
419	-0.40	0.08	0.03	-0.02	459	-0.39	-0.03	0.14	0.01	509	-0.38	-0.17	0.26	0.06
410	-0.40	0.09	0.02	-0.02	460	-0.39	-0.02	0.13	0.01	510	-0.38	-0.16	0.25	0.06
411	-0.40	0.10	0.01	-0.02	461	-0.39	-0.01	0.12	0.00	511	-0.38	-0.15	0.25	0.05
412	-0.40	0.11	0.00	-0.02	462	-0.39	0.00	0.11	0.00	512	-0.38	-0.14	0.24	0.05
413	-0.40	0.12	-0.01	-0.02	463	-0.39	0.01	0.10	-0.00	513	-0.38	-0.13	0.23	0.05
414	-0.40	0.13	-0.02	-0.02	464	-0.39	0.02	0.09	-0.01	514	-0.38	-0.12	0.22	0.04
415	-0.40	0.14	-0.03	-0.03	465	-0.39	0.03	0.08	-0.01	515	-0.38	-0.11	0.21	0.04
416	-0.40	0.15	-0.04	-0.03	466	-0.39	0.04	0.07	-0.01	516	-0.38	-0.10	0.20	0.03
417	-0.40	0.16	-0.04	-0.03	467	-0.39	0.05	0.06	-0.01	517	-0.38	-0.09	0.19	0.03
418	-0.40	0.17	-0.05	-0.03	468	-0.39	0.06	0.05	-0.01	518	-0.38	-0.08	0.18	0.03
419	-0.40	0.18	-0.06	-0.03	469	-0.39	0.07	0.04	-0.02	519	-0.38	-0.07	0.17	0.02
420	-0.40	0.19	-0.07	-0.03	470	-0.39	0.08	0.03	-0.02	520	-0.38	-0.06	0.16	0.02
421	-0.40	0.20	-0.08	-0.03	471	-0.39	0.09	0.02	-0.02	521	-0.38	-0.05	0.15	0.02
422	-0.40	0.21	-0.09	-0.03	472	-0.39	0.10	0.01	-0.02	522	-0.38	-0.04	0.14	0.01
423	-0.40	0.22	-0.10	-0.03	473	-0.39	0.11	0.00	-0.02	523	-0.38	-0.03	0.13	0.01
424	-0.40	0.23	-0.11	-0.03	474	-0.39	0.12	-0.01	-0.02	524	-0.38	-0.02	0.13	0.01
425	-0.40	0.24	-0.12	-0.02	475	-0.39	0.13	-0.02	-0.02	525	-0.38	-0.01	0.12	0.00
426	-0.40	0.25	-0.13	-0.02	476	-0.39	0.14	-0.03	-0.02	526	-0.38	0.00	0.11	0.00
427	-0.40	0.26	-0.14	-0.02	477	-0.39	0.15	-0.04	-0.03	527	-0.38	0.01	0.10	-0.00
428	-0.40	0.27	-0.14	-0.02	478	-0.39	0.16	-0.05	-0.03	528	-0.38	0.02	0.09	-0.01
429	-0.40	0.28	-0.15	-0.02	479	-0.39	0.17	-0.06	-0.03	529	-0.38	0.03	0.08	-0.01
430	-0.40	0.29	-0.16	-0.02	480	-0.39	0.18	-0.06	-0.03	530	-0.38	0.04	0.07	-0.01
431	-0.39	-0.31	0.37	0.12	481	-0.39	0.19	-0.07	-0.03	531	-0.38	0.05	0.06	-0.01
432	-0.39	-0.30	0.36	0.12	482	-0.39	0.20	-0.08	-0.03	532	-0.38	0.06	0.05	-0.01
433	-0.39	-0.29	0.36	0.11	483	-0.39	0.21	-0.09	-0.03	533	-0.38	0.07	0.04	-0.02
434	-0.39	-0.28	0.35	0.11	484	-0.39	0.22	-0.10	-0.02	534	-0.38	0.08	0.03	-0.02
435	-0.39	-0.27	0.34	0.10	485	-0.39	0.23	-0.11	-0.02	535	-0.38	0.09	0.02	-0.02
436	-0.39	-0.26	0.34	0.10	486	-0.39	0.24	-0.12	-0.02	536	-0.38	0.10	0.01	-0.02
437	-0.39	-0.25	0.33	0.10	487	-0.39	0.25	-0.13	-0.02	537	-0.38	0.11	-0.00	-0.02
438	-0.39	-0.24	0.32	0.09	488	-0.39	0.26	-0.13	-0.02	538	-0.38	0.12	-0.01	-0.02
539	-0.38	0.13	-0.02	-0.02	553	-0.38	0.27	-0.14	-0.02	567	-0.37	-0.25	0.32	0.10
540	-0.38	0.14	-0.03	-0.02	554	-0.38	0.28	-0.15	-0.02	568	-0.37	-0.24	0.31	0.09
541	-0.38	0.15	-0.04	-0.02	555	-0.38	0.29	-0.16	-0.02	569	-0.37	-0.23	0.30	0.09
542	-0.38	0.16	-0.05	-0.02	556	-0.38	0.30	-0.17	-0.01	570	-0.37	-0.22	0.30	0.08
543	-0.38	0.17	-0.06	-0.02	557	-0.38	0.31	-0.17	-0.01	571	-0.37	-0.21	0.29	0.08
544	-0.38	0.18	-0.07	-0.03	558	-0.38	0.32	-0.18	-0.01	572	-0.37	-0.20	0.28	0.08
545	-0.38	0.19	-0.07	-0.02	559	-0.37	-0.33	0.37	0.13	573	-0.37	-0.19	0.27	0.07
546	-0.38	0.20	-0.08	-0.02	560	-0.37	-0.32	0.36	0.12	574	-0.37	-0.18	0.26	0.07
547	-0.38	0.21	-0.09	-0.02	561	-0.37	-0.31	0.35	0.12	575	-0.37	-0.17	0.26	0.06
548	-0.38	0.22	-0.10	-0.02	562	-0.37	-0.30	0.35	0.12	576	-0.37	-0.16	0.25	0.06
549	-0.38	0.23	-0.11	-0.02	563	-0.37	-0.29	0.34	0.11	577	-0.37	-0.15	0.24	0.05
550	-0.38	0.24	-0.12	-0.02	564	-0.37	-0.28	0.34	0.11	578	-0.37	-0.14	0.23	0.05
551	-0.38	0.25	-0.13	-0.02	565	-0.37	-0.27	0.33	0.11	579	-0.37	-0.13	0.22	0.05
552	-0.38	0.26	-0.13	-0.02	566	-0.37	-0.26	0.32	0.10	580	-0.37	-0.12	0.21	0.04

From the complete table containing 7822 values, it was found that $-0.21 \le \rho_1 \le 0.37$ and $-0.03 \le \rho_2 \le 0.17$ for model (2.1).

CONCLUSIONS

We have considered a method of differentiating between pure diagonal bilinear model of order two and moving average of order two based on the properties of their autocorrelation functions. Using the second derivative test and tabular method, the minimum and maximum values of the autocorrelation coefficients of the two models are obtained as follows:

$-0.71 \le \rho_1 \le 0.71$ and $-0.50 \le \rho_2 \le 0.50$ for MA(2) process.

and

$-0.21 \le \rho_1 \le 0.37$ and $-0.03 \le \rho_2 \le 0.17$ PDB(2) process.

Hence, the two models are only comparable when $-0.21 \le \rho_1 \le 0.37$ and $-0.03 \le \rho_2 \le 0.17$ as shown in Figure 2.

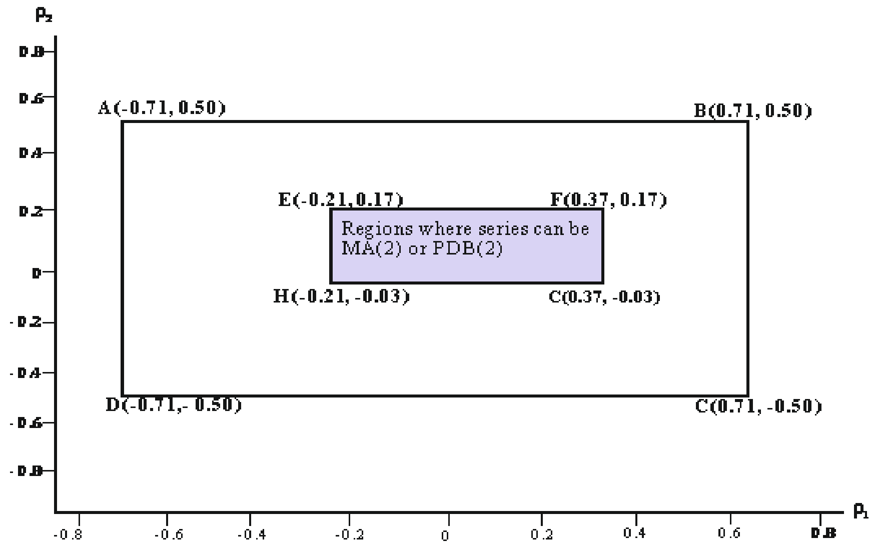

Figure 2: Determination of Region for Comparing MA (2) And PBD (2)

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